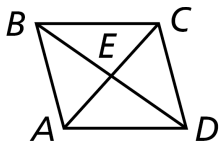
Attendance Problems.

1. Find AB for A(-3, 5) and B(1, 2).

2. Find the slope of $\overline{JK}$ for J(-4, 4) and K(3, -3).

ABCD is a parallelogram.

3. Why is $\measuredangle ABC \cong \measuredangle CDA$?

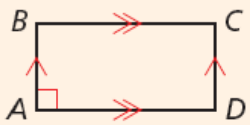
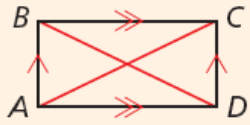


4. Why is $\measuredangle AEB \cong \measuredangle CED$?

I can prove that a given quadrilateral is a rectangle, rhombus, or square.

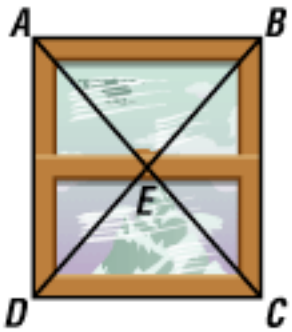
Common Core: **CC.9-12.G.CO.11** Prove theorems about parallelograms.

Theorems Conditions for Rectangles

	THEOREM	EXAMPLE
6-5-1	If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle. (▱ with one rt. ∠ → rect.)	
6-5-2	If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (▱ with diags. ≅ → rect.)	 $\overline{AC} \cong \overline{BD}$

Refer to video example 1.

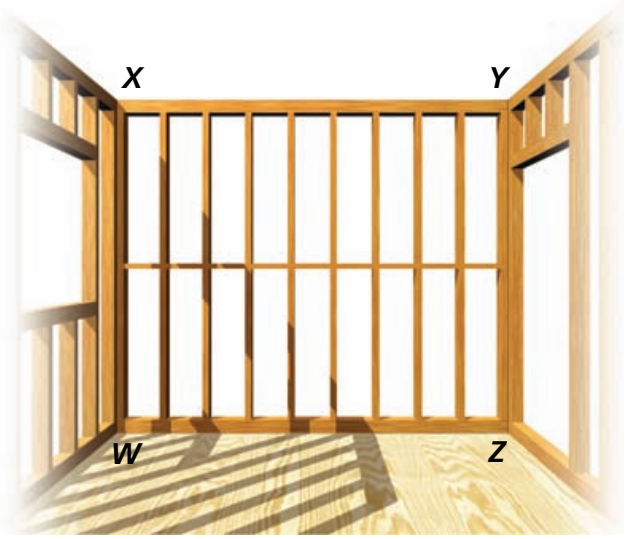
A carpenter built a wood frame for a window so that $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$, and $\overline{AD} \cong \overline{BC}$. Why must the frame be a rectangle?



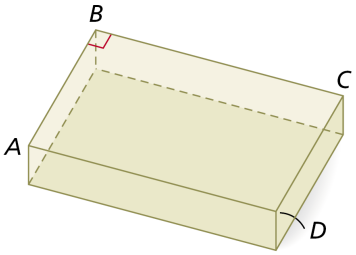
1 Carpentry Application

A contractor built a wood frame for the side of a house so that $\overline{XY} \cong \overline{WZ}$ and $\overline{XW} \cong \overline{YZ}$. Using a tape measure, the contractor found that $XZ = WY$. Why must the frame be a rectangle?

Both pairs of opposite sides of $WXYZ$ are congruent, so $WXYZ$ is a parallelogram. Since $XZ = WY$, the diagonals of $\square WXYZ$ are congruent. Therefore the frame is a rectangle by Theorem 6-5-2.

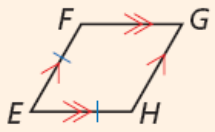
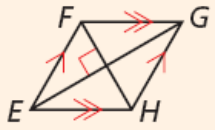
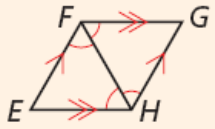


Example 1. A manufacturer builds a mold fro a desktop so that $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}, \& \; m\angle ABC = 90°$. Why must ABCD be a rectangle?



5. Guided Practice. A carpenter's square can be used to test that an angle is a right angle. How could the contractor use a carpenter's square to check that the frame is a rectangle?

Theorems — Conditions for Rhombuses

	THEOREM	EXAMPLE
6-5-3	If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. (▱ with one pair cons. sides ≅ → rhombus)	
6-5-4	If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (▱ with diags. ⊥ → rhombus)	
6-5-5	If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (▱ with diag. bisecting opp. ∠ → rhombus)	

Caution

In order to apply Theorems 6-5-1 through 6-5-5, the quadrilateral must be a parallelogram.

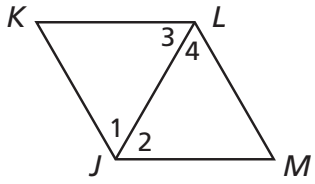
Remember!

You can also prove that a given quadrilateral is a rectangle, rhombus, or square by using the definitions of the special quadrilaterals.

Theorem 6-5-5

Given: $JKLM$ is a parallelogram.
$\overline{JL}$ bisects $\angle KJM$ and $\angle KLM$.
Prove: $JKLM$ is a rhombus.

Proof:



Statements	Reasons
1. $JKLM$ is a parallelogram. $\overline{JL}$ bisects $\angle KJM$ and $\angle KLM$.	1. Given
2. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$	2. Def. of $\angle$ bisector
3. $\overline{JL} \cong \overline{JL}$	3. Reflex. Prop. of $\cong$
4. $\triangle JKL \cong \triangle JML$	4. ASA *Steps 2, 3*
5. $\overline{JK} \cong \overline{JM}$	5. CPCTC
6. $JKLM$ is a rhombus.	6. $\square$ with one pair cons. sides $\cong \rightarrow$ rhombus

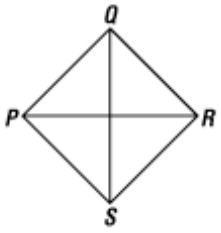
Refer to video example 2.

A.

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

Given:
$\overline{PQ} \cong \overline{SR}$, $\overline{PS} \cong \overline{QR}$, $\overline{PQ} \perp \overline{QR}$, and $\overline{PR} \perp \overline{QS}$.

Conclusion:
$PQRS$ is a square.

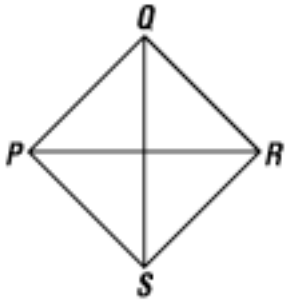


B.

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

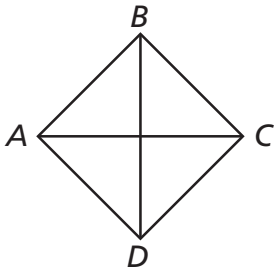
Given:
$\overline{PR} \perp \overline{QS}$

Conclusion:
PQRS is a rhombus.



2 Applying Conditions for Special Parallelograms

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



A **Given:** $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{BD}$

Conclusion: *ABCD* **is a square.**

Step 1 Determine if *ABCD* is a parallelogram.

$\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$ — *Given*

ABCD is a parallelogram. — *Quad. with opp. sides ≅ → ▱*

Step 2 Determine if *ABCD* is a rectangle.

$\overline{AD} \perp \overline{DC}$, so $\angle ADC$ is a right angle. — *Def. of ⊥*

ABCD is a rectangle. — *▱ with one rt. ∠ → rect.*

Step 3 Determine if *ABCD* is a rhombus.

$\overline{AC} \perp \overline{BD}$ — *Given*

ABCD is a rhombus. — *▱ with diags. ⊥ → rhombus*

Step 4 Determine if *ABCD* is a square.

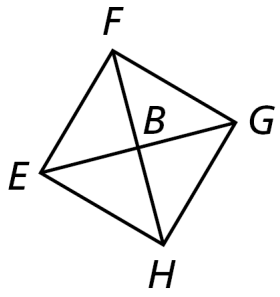
Since *ABCD* is a rectangle and a rhombus, it has four right angles and four congruent sides. So *ABCD* is a square by definition. The conclusion is valid.

B **Given:** $\overline{AB} \cong \overline{BC}$

Conclusion: *ABCD* **is a rhombus.**

The conclusion is not valid. By Theorem 6-5-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. To apply this theorem, you must first know that *ABCD* is a parallelogram.

<u>Example 2</u>. Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid._



A. Given: $\overline{EF} \cong \overline{FG}$ & $\overline{EG} \perp \overline{FH}$

Conclusion: EFGH is rhombus.

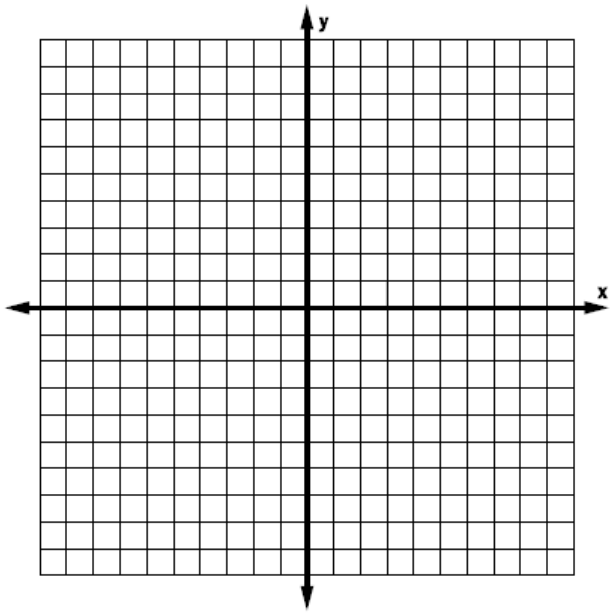
B. Given: $\overline{EB} \cong \overline{BG}, \overline{FB} \cong \overline{BH}, \overline{EG} \cong \overline{FH}, \triangle EBF \cong \triangle EBH$

Conclusion: EFGH is a square.

Refer to video example 3.

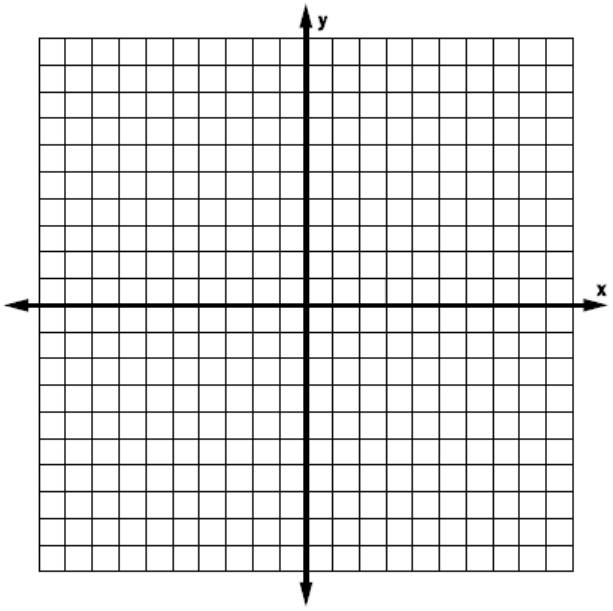
Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

A(0, 2), *B*(2, 7), *C*(7, 5), *D*(5, 0)

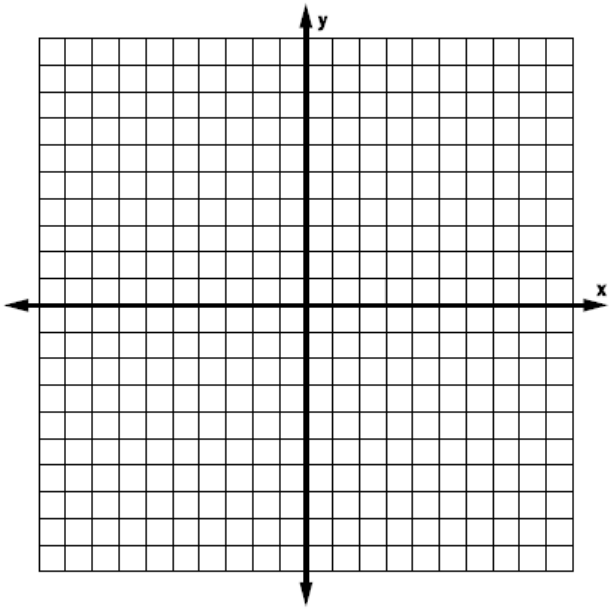


<u>Guided Practice</u>. Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

7. $K(-5, -1)$, $L(-2, 4)$, $M(3, 1)$, $N(0, -4)$



8. $P(-4, 6)$, $Q(2, 5)$, $R(3, -1)$, $S(-3, 0)$



6-5 Conditions for Special Parallelograms *(p 434)* 6-10, 14, 16, 17, 27, 29, 33.